

Constraints on the relativistic mean field of Δ -isobar in nuclear matter

D.S. Kosov¹, C. Fuchs¹, B.V. Martemyanov^{1,2}, Amand Faessler¹

¹ *Institut für Theoretische Physik, Universität Tübingen, Auf der Morgenstelle 14, D-72076 Tübingen, Germany*

² *Institute for Theoretical and Experimental Physics, B.Chermushkinskaya 25, 117259 Moscow, Russia*

Abstract

The effects of the presence of Δ -isobars in nuclear matter are studied in the framework of relativistic mean-field theory. The existence of stable nuclei at saturation density imposes constraints on the Δ -isobar self-energy and thereby on the mean-field coupling constants of the scalar and vector mesons with Δ -isobars. The range of possible values for the scalar and vector coupling constants of Δ -isobars with respect to the nucleon coupling is investigated and compared to recent predictions of QCD sum-rule calculations.

Key words: Δ -isobar, Quantum Hadron Dynamics, QCD sum-rule

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The role of the Δ -isobars in nuclear matter has been discussed in various investigations in the framework of effective hadron field theories, i.e. Quantum Hadron Dynamics (QHD) [1–3]. In the framework of QHD [4] nucleons as well as Δ -isobars interact with the surrounding nuclear medium by the exchange of effective scalar σ and vector V^μ -mesons. However, the coupling strengths of the respective baryon-meson vertices are supposed to be different for nucleons and deltas. A lot of interesting physical observable depend crucially on the choice of the respective coupling strengths [1–3]. In the framework of effective QHD the coupling constants for the nucleon-meson vertices are effective quantities which are normally adjusted to the nuclear matter saturation properties [4]. They can, however, be also taken from different sources, e.g. from chiral models [5] or from Dirac-Brueckner-Hartree-Fock calculations [6]. There is up to now no definite information about the scalar and vector delta coupling strengths.

Recent studies based on the QCD finite density sum-rule [7] have shown that the Δ vector self-energy is considerably weaker than that of the nucleon, while its scalar self-energy is somewhat stronger. However, the QCD sum-rule predictions for the scalar self-energy are sensitive to the unknown density dependence of four-quark condensates and due to this there is no reliable information about the coupling constant of the Δ -isobar with scalar mesons. Thus phenomenological constraints on scalar and vector coupling constants of Δ -isobar are of general interest, in particular because they give new restrictions on the dependence of four-quark condensates on the nucleon density. Further such considerations show up the range for possible values of the respective vertices. This is also of importance for phenomenological applications such as, e.g., heavy ion collisions where the decay and rescattering of Δ -isobars are the prominent sources of meson production [8,9]. In relativistic heavy ion collisions the excess of deltas can reach values comparable to the saturation density of nuclear matter (so called resonance matter) [8]. In the theoretical description within transport models the Δ -isobars are commonly treated with the same coupling strengths as the nucleons, i.e. the scalar and vector components of the relativistic mean field are assumed to be identical for nucleons and Δ -isobars [9,10]. This has of course influence on the delta dynamics as well as on the description of inelastic collisions, e.g., the thresholds for the production of mesons would be affected [10]. In order to use, e.g., pions or kaons as a source of information on the hot and compressed phase of nuclear matter the understanding of the properties of Δ -isobars in the nuclear medium appears to be indispensable.

In the present work we investigate the mixed phase of nuclear and Δ -matter within the framework of effective QHD. The model system we are considering consists of nucleons Ψ_N , Δ -isobars which are treated as a Rarita-Schwinger particle $\Psi_{\Delta\alpha}$ [11], effective scalar σ and vector V^μ meson fields which are both isoscalars. The inclusion of these mesons is sufficient for the description of symmetric nuclear matter with a vanishing isovector density and also in the

mixed phase where the total (zero) isospin should be conserved. We adopt the extension of the Walecka model [4] by including Δ -isobars as an additional degree of freedom in the Lagrangian [1]

$$\mathcal{L} = \mathcal{L}_f - g_v \bar{\Psi}_N \gamma^\mu V_\mu \Psi_N - g'_v \bar{\Psi}_\Delta^\alpha \gamma^\mu V_\mu \Psi_{\Delta\alpha} + g_\sigma \bar{\Psi}_N \sigma \Psi_N + g_\sigma \bar{\Psi}_\Delta^\alpha \sigma \Psi_{\Delta\alpha} + U(\sigma) \quad . \quad (1)$$

Here \mathcal{L}_f collects a free Lagrangian of the baryon fields Ψ_N , $\Psi_{\Delta\alpha}$ and the V^μ , σ meson fields. The interaction terms of the nucleons and Δ -isobars with the meson fields are explicitly given in Eq. (1). The respective coupling constants of nucleons g_σ (g_v) and deltas g'_σ (g'_v) with the scalar (vector) mesons can be different in magnitude. The phenomenological self-interaction of the scalar field $U(\sigma)$ specified as [12]

$$U(\sigma) = -\frac{1}{3}B\sigma^3 - \frac{1}{4}C\sigma^4 \quad (2)$$

introduces an additional non-linear density dependence into the model which improves the description of nuclear matter bulk properties. The respective parameters taken from Ref. [13] yield a saturation density $\rho_0 = 0.167 fm^{-3}$, a binding energy of -15.8 MeV and a compressibility of 290 MeV. To keep the model as general as possible no restrictions are imposed on the Δ -meson vertices from the beginning, i.e., g'_σ and g'_v are treated as free parameters. The range of their possible values is finally obtained by the variation of $g'_{\sigma,v}$ and the consideration of the resulting equation of state.

In the mean field approximation the meson fields in Eq. (1) are given by their classical expectation values over the ground state of the nuclear system. Thus one obtains the Lagrangian density in the mean-field approximation. The Dirac field equations follow from mean field Lagrangian density as

$$[i\gamma^\mu \partial_\mu - g_v \gamma^\mu V_\mu - M_N^*] \Psi_N(x) = 0 \quad (3)$$

$$[i\gamma^\mu \partial_\mu - g'_v \gamma^\mu V_\mu - M_\Delta^*] \Psi_\Delta^\alpha(x) = 0 \quad . \quad (4)$$

The effective masses are given as

$$M_N^* = M_N - g_\sigma \sigma \quad , \quad M_\Delta^* = M_\Delta - g'_\sigma \sigma \quad (5)$$

where M_N, M_Δ stand for the bare nucleon and Δ -masses. In infinite nuclear matter the mean field approximation yields the scalar σ and vector $V_\mu = \delta_{\mu 0} V_0$ meson fields which are time and space independent and directly proportional to the respective source terms, i.e.

$$V_0 = \frac{g_v}{m_v^2} \rho_B(N) + \frac{g'_v}{m_v^2} \rho_B(\Delta) \quad (6)$$

$$m_\sigma^2 \sigma + B\sigma^2 + C\sigma^3 = g_\sigma \rho_s(N) + g'_\sigma \rho_s(\Delta) \quad (7)$$

with $\rho_B(N), \rho_s(N), \rho_B(\Delta), \rho_s(\Delta)$ being the nucleon and Δ -isobar baryon and scalar densities. The equation for the effective nucleon mass M_N^* is given by

$$M_N^* = M_N - \frac{g_\sigma^2}{m_\sigma^2} \rho_s(N) - \frac{g_\sigma g'_\sigma}{m_\sigma^2} \rho_s(\Delta) + \frac{B}{g_\sigma m_\sigma^2} (M_N - M_N^*)^2 + \frac{C}{g_\sigma^2 m_\sigma^2} (M_N - M_N^*)^3 \quad (8)$$

which has to be solved selfconsistently. The effective Δ -mass can be found as a function of M_N^* and the scalar coupling constant g'_σ, g_σ from Eq. (5).

Chemical stability of the nucleons and Δ -isobars requires that the chemical potentials (Fermi energies) are equal:

$$E_F(N) = E_F(\Delta) , \quad (9)$$

with

$$E_F(N) = g_v V_0 + \sqrt{k_F(N)^2 + M_N^{*2}} \quad (10)$$

$$E_F(\Delta) = g'_v V_0 + \sqrt{k_F(\Delta)^2 + M_\Delta^{*2}} . \quad (11)$$

The Fermi momentum $k_F(\Delta)$ of the Δ -isobar matter is determined from Eq.(9) and $k_F(\Delta)$ is related to the density by usual relation, thereby the Δ -isobars density is governed by the requirement of a chemical equilibrium Eq.(9).

In Fig. 1 we show the equation of state, i.e., the energy per baryon as a function of baryon density for different ratios of scalar and vector coupling constants of the Δ over those of the nucleon $r_s = g'_\sigma/g_\sigma$ and $r_v = g'_v/g_v$. It is found that the presence of Δ -isobars becomes energetically favorable at higher densities ($\rho > 2\rho_0$) where the mixed Δ -nucleon phase appears. The second minimum corresponds to new metastable state of nuclear matter as already found in Ref. [1].

We obtain the first constraint on the delta coupling constants, i.e., on ratios r_s and r_v by the requirement that the second minimum should lie above the saturation energy of normal nuclear matter (a). This means that in the mixed Δ -nucleon phase only a metastable state can occur. An additional restriction on the coupling constant can be obtained if one requires that there are no

Δ -isobars present at saturation density. In other words, the inclusion of a Δ -isobar field in the Lagrangian (1) should not effect the value for the binding energy of nuclear matter at saturation density, i.e. $k_F(\Delta) = 0$ at saturation density (b). This requirement which is based on the fact that there are no Δ -isobars in the ground state of finite nuclei imposes a "soft" restriction on the coupling constants [2]

$$r_s \leq 0.82r_v + 0.71 \quad . \quad (12)$$

Recent QCD finite density sum-rule calculations [7] have shown that there exists a larger net attraction for a Δ -isobar than for a nucleon in the nuclear medium. Thus we can further restrict our consideration to the cases where $r_s \geq 1$ and $r_v \leq 1$ (c).

In Fig.2 we show the "window" for the possible values of ratios which are allowed under the restrictions (a-c). The constraints (a,b) (under the condition (c)) can thereby be summarized as

$$r_s \leq 1.01r_v + 0.38 \quad (13)$$

which imposes a more restrictive constraint on the ratios than Eq. (12).

The QCD sum-rule calculations predict [7] that the Δ -isobar vector coupling is two times smaller ($r_v \sim 0.4 - 0.5$) than the corresponding nucleon coupling and the magnitude of the Δ -isobar scalar coupling is larger than that for nucleon ($r_s \sim 1.3$). We checked these ratios between the Δ -isobar and nucleon coupling constants by our phenomenological model and we found in this case that the depth of the second minimum is of more than one order of magnitude larger ($280MeV$) than the minimum at saturation density. This means that for the QCD predictions of Ref. [7] the real ground state of nuclear matter would occur to be a Δ -isobar matter at a density of about $\sim 3\rho_0$ and a binding energy of $\sim 280MeV$.

To summarize we have investigated the properties of Δ -isobars in nuclear matter within the framework of Quantum Hadron Dynamics. Based on the constraints that there are no Δ -isobars present at saturation density, the appearance of Δ -isobars at higher density leads to a metastable state which is not the ground state of nuclear matter and that the scalar interaction is more attractive and the vector interaction is less repulsive for deltas than for nucleons we have been able to derive a window for possible values of mean field scalar and vector coupling constants for the delta. It turned out that present QCD sum-rule predictions do not fall into that window and seem to yield too strong deviations of the Δ -isobar self-energy from the nucleon self-energy in the nuclear medium.

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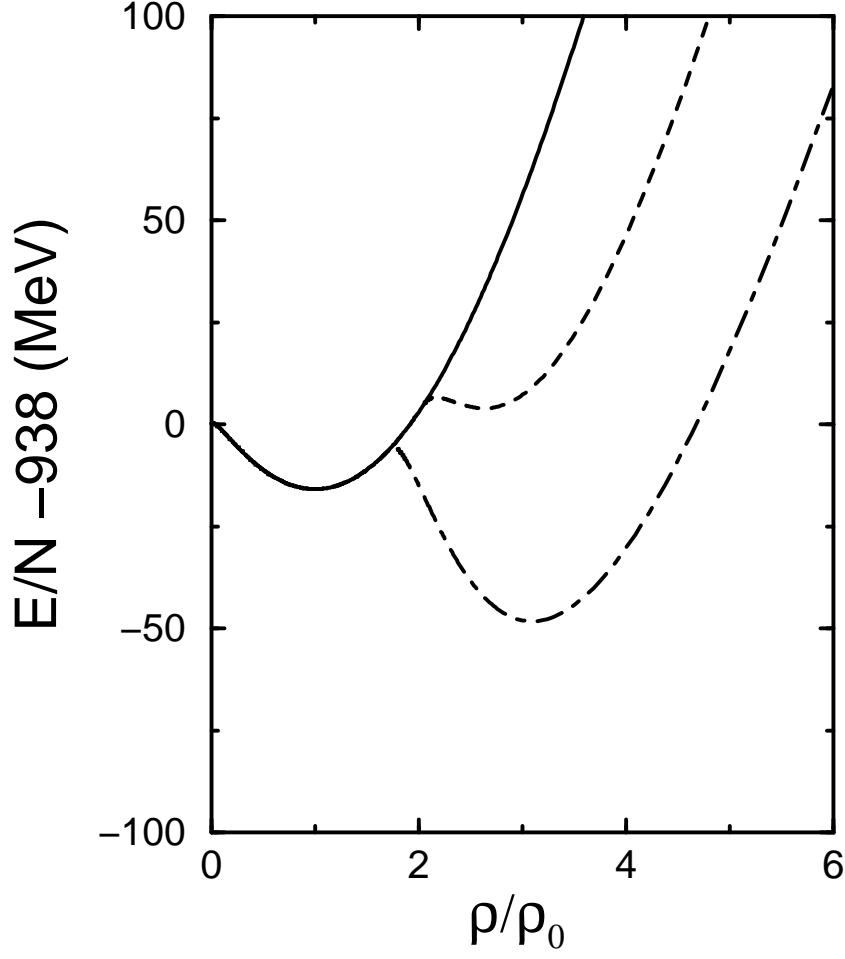


Fig. 1. Equation of state, i.e. binding energy per baryon versus density for nuclear matter in the mixed nucleon and Δ -isobar phase. The calculation are performed for various values of r_s and r_v ($r_s = 1.35$, $r_v = 1$ for the dashed curve, and $r_s = 1.35$, $r_v = 0.9$ for the dash-dotted curve). The solid curve shows the nuclear matter equation of state without Δ -isobars.

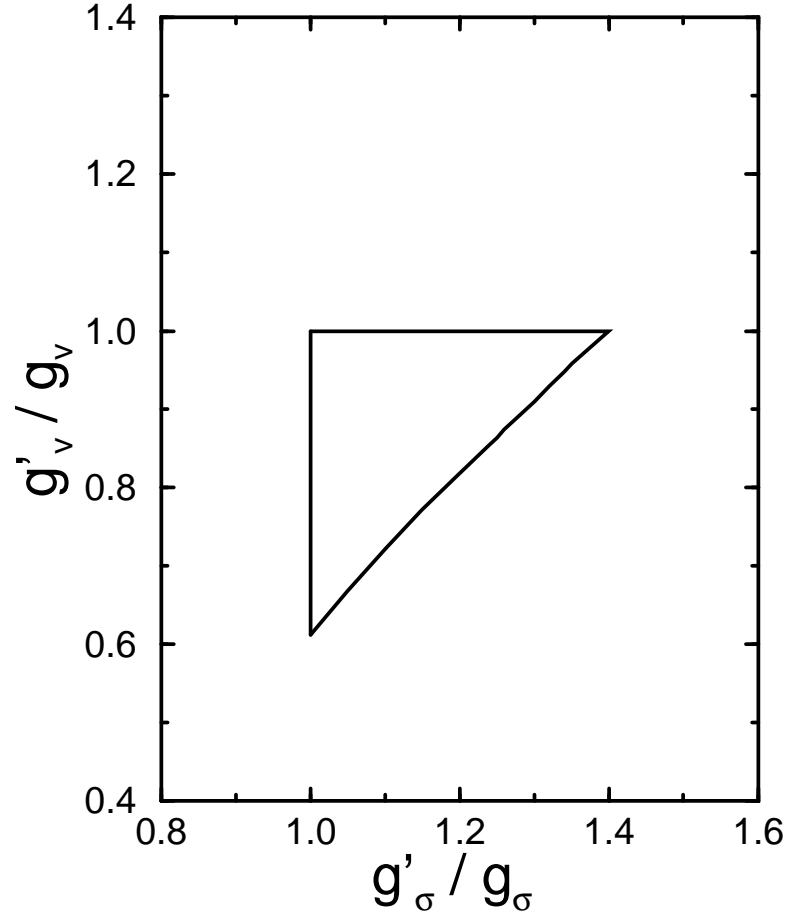


Fig. 2. Values of ratios of scalar and vector coupling constant for a Δ -isobar to that for a nucleon which are allowed under the assumption: (a) The mixed Δ -nucleon state is metastable, i.e. the binding energy at the second minimum is smaller than in ground state nuclear matter. (b) There are no deltas present at the saturation density. (c) The scalar mean field is more attractive and the vector potential is less repulsive for deltas than for nucleons.